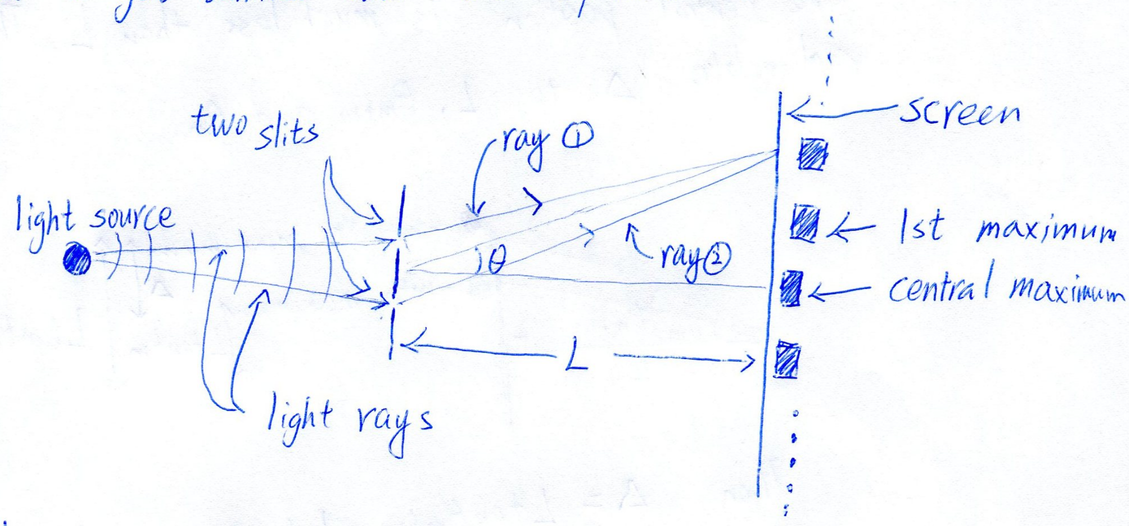
Solution by Leung Hing Shing.

Aim: To calculate the wavelength of light using Young's two-slit experiment setup, and study the effects of changing double-slit experiment parameters.

Double-slit experiment is one of the most important experiment in the history of physics. And also, you need to understand the Young's double-slit experiment of light before understanding the particle-wave duality using Young's experiment. Thus, it is important for you to get familiar with the experiment.



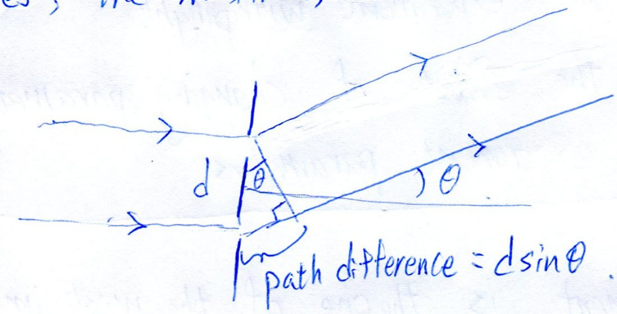
d: slit separation   L: slit-screen seperation. $(L \gg d)$.

θ_n: angle at which the n-th maximum is located.

λ: the wavelength of the light.

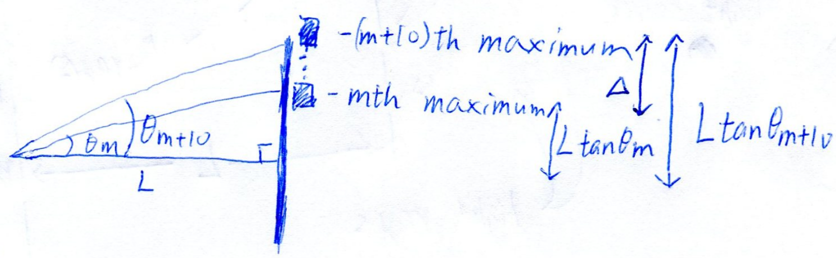
Fig. 1. The Young's double-slit experiment setup.

Constructive interferences occur when the path differences $(d\sin\theta)$ are integral multiple of λ $(d\sin\theta_n = n\lambda)$, leaving bright fringes, the maxima, on the screen. (P.2)



Given seperation between the m-th maximum and the (m+10)-th maximum on the screen $\Delta = 7.5$ mm, $d = 0.4$ mm and $L = 600$ mm. We want to calculate λ.

① Assume that the seperation of the (m+10)-th maximum and the central position is much less than L. Then, we can relate Δ to L, θ_{m+10} and θ_m.



Then $\Delta = L\tan\theta_{m+10} - L\tan\theta_m$

$$\approx L\sin\theta_{m+10} - L\sin\theta_m \quad \left(\begin{array}{r}\text{since } L\tan\theta \ll L \\ \tan\theta \ll 1 \\ \sin\theta \approx \tan\theta\end{array}\right)$$

② State the conditions for the maxima to appear.

$d \sin\theta_m = m\lambda$

$d \sin\theta_{m+10} = (m+10)\lambda$.

$\Rightarrow \sin\theta_{m+10} - \sin\theta_m = \frac{10\lambda}{d}$.

③ calculate λ using $\Delta = L\sin\theta_{m+10} - L\sin\theta_m$

and $\sin\theta_{m+10} - \sin\theta_m = \frac{10\lambda}{d}$.

Then $\Delta = L\left(\frac{10\lambda}{d}\right)$

$$\Rightarrow \lambda = \frac{d\Delta}{10L} = \frac{(0.4\text{ mm})(7.5\text{ mm})}{(10)(600\text{ mm})}$$

$$= 5\times10^{-4}\text{ mm} = 5\times10^{-7}\text{ m}.$$

This is green light.

(a) Using $\Delta = L\left(\frac{10\lambda}{d}\right)$ derived above,

and assume $L\tan\theta \ll L$.

$$\Delta = (600\text{ mm})\left(\frac{(10)(5\times10^{-4}\text{ mm})}{(0.2\text{ mm})}\right)$$

$$= 15\text{ mm}.$$

(b). $$\Delta = (800\text{mm})\left(\frac{(10)(5\times10^{-4}\text{mm})}{(0.4\text{mm})}\right)$$

$$= 10\text{mm}.$$

(c). $$\Delta = (600\text{mm})\left(\frac{(10)(7\times10^{-4}\text{ mm})}{(0.4\text{ mm})}\right)$$

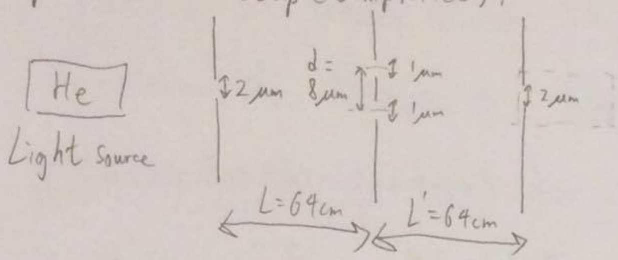
$$= 10.5\text{ mm}.$$

The table below shows the effects of parameters on Δ P.4

parameters	Δ
$d\downarrow$ from 0.4 mm to 0.2 mm	$\uparrow$ from 7.5 mm to 15.0 mm
$L\uparrow$ from 600 mm to 800 mm	$\uparrow$ from 7.5 mm to 10 mm
$\lambda\uparrow$ from 5×10^{-4} mm to 7×10^{-4} mm	$\uparrow$ from 7.5 mm to 10.5 mm

SQ6) Reference ; O. Carnal and J. Mlynek, Phys. Rev. Lett. 66, 2689 (1991) (P.5)

Experimental setup (Simplified) ;



stepper motor detector

step = 1.88 μm

* Inside vacuum chamber

$p \approx 5\times10^{-7}$ mbar

Ambient pressure ≈ 1 bar

Some data reported ;

$\boxed{T = 295K :}$ $\lambda_{dB} = 0.56\text{Å}$

Theoretical value of distance between two maxima = 4.48 μm (4.5 μm)

Measured value = 4.5 ± 0.6 μm

$\boxed{T = 83K :}$ $\lambda_{dB} = 1.03\text{Å}$

Theoretical value of distance between two maxima = 8.24 μm (8.2 μm)

Measured value = 8.4 ± 0.8 μm

Using perodic grating (~4 μm wide) ;

Measured value = 7.7 ± 0.5 μm

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① How to get $d_x = \frac{L\lambda_{dB}}{d}$?

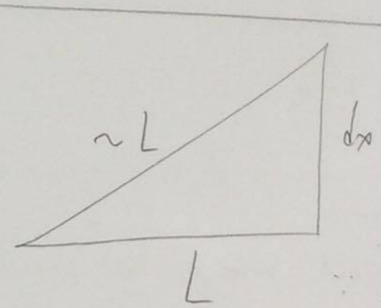
> The fact that the distance between two maxima is much shorter that the distance between the slit and detector !!

$\sin\theta = \frac{d_x}{L}$

$d \sin\theta = n\lambda$ (well known, always true)

$\frac{d_x}{L} = \frac{n\lambda_{dB}}{d}$

$d_x = \frac{L\lambda_{dB}}{d}$ (Let $n = 1$)



$\because L^2 + d_x^2 \approx L^2$

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② Corresponding λ_{dB} in different setup ?

perodic grating ;

$d_x = 7.7 \pm 0.5$ μm

$\lambda_{dB} = 1.03\text{Å}$ (expected)

$\lambda_{dB} \approx 0.963\text{Å}$ (calculated)

two slits ;

$d_x = 4.5 \pm 0.6$ μm

$\lambda_{dB} = 0.56\text{Å}$ (expected)

$\lambda_{dB} \approx 0.563\text{Å}$ (calculated)

③ Estimate λ_{dB} ?

$\langle v \rangle$: Mean velocity $= \frac{2}{\sqrt{\pi}}\sqrt{\frac{2RT}{M}}$

p : Momentum $= mv$

De Broglie relation : $p = \frac{h}{\lambda_{dB}}$

R : gas constant $\approx 8.31\ kg\,m^2 s^{-2} K^{-1} mol^{-1}$

M : molar mass of helium $\approx 0.004\ kg\,mol^{-1}$

m : mass of a helium atom $\approx \frac{0.004}{6.02\times10^{23}}\ kg$

h : Planck's constant $= 6.63\times10^{-34}\ Js$

$*\ 1J = 1N\cdot m = \frac{kg\cdot m}{s^2}\ m$

At $T = 295K$,

$\langle v \rangle = \frac{2}{\sqrt{\pi}}\sqrt{\frac{2RT}{M}}$

$\approx 1250\ ms^{-1}$

$p \approx 8.3\times10^{-24}\ kgms^{-1}$

$\lambda'_{dB} = \frac{h}{p}$

$= \frac{6.63\times10^{-34}\ Js}{8.3\times10^{-24}\ kgms^{-1}}$

$\approx 0.799 Å$

$\Delta\lambda_{dB} = |\lambda_{dB} - \lambda'_{dB}|$

$\approx 0.24 Å$

At $T = 83K$,

$\langle v \rangle \approx 663 ms^{-1}$

$p \approx 4.40\times10^{-24}\ kgms^{-1}$

$\lambda'_{dB} \approx 1.51 Å$

$\Delta\lambda_{dB} = |\lambda_{dB} - \lambda'_{dB}|$

$\approx 0.48 Å$

Source of errors:

① The constant value is not precise enough.

② Mean speed is the major error of this estimation. It will be better to consider most probable speed.

SQ.7

Aim: to understand the wave properties of a wave system, including the fundamental frequency ; the fundamental wavelength and the wave propagation

(a). We want to check whether the standard wave form $\sim \cos(kx-\omega t)$ satisfies the EM wave equation.

wavenumber (k), angular frequency (ω)

① Searching for the EM wave equation on the internet

The wave equation is $\boxed{\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}}$, where $E(x,t)$ is the electric field of the EM wave at (x,t), and c is the speed of light in vacuum. We consider a wave propagating in x-direction only, so <u>this equation describe the propagation of EM wave in the x-direction</u>.

② Trying to test the standard wave form wave if it satisfies the wave equation.

The stardard waveform is $E = E_0 \cos(kx-\omega t)$

$$\therefore \frac{\partial E}{\partial x} = E_0\left(-\sin(kx-\omega t)\right)(k)$$

$$\frac{\partial^2 E}{\partial x^2} = E_0\left[\left(-\cos(kx-\omega t)(k)\right)\right](k) = -k^2 E_0 \cos(kx-\omega t)$$

$$= -k^2 E$$

$$\frac{\partial E}{\partial t} = E_0\left[-\sin(kx-\omega t)\right](-\omega) = E_0 \sin(kx-\omega t)(\omega).$$

$$\frac{\partial^2 E}{\partial t^2} = E_0[\cos(kx-\omega t)\ (-\omega)](\omega) = -\omega^2 E_0 \cos(kx-\omega t)$$
$$= -\omega^2 E$$

Therefore, we have

$$\boxed{\begin{aligned} \frac{\partial^2 E}{\partial t^2} &= -\omega^2 E \\ \frac{\partial^2 E}{\partial x^2} &= -k^2 E \end{aligned}}$$

An addition law required is the "ck law", which is $\boxed{\omega = ck}$ In our department, we have a professor, teaching a EM course, called $\boxed{C}$hi-$\boxed{K}$wong $\boxed{Law}$(羅志光). He always reminds me of this "ck law".

Thanks for ck law, We can now finish the question.

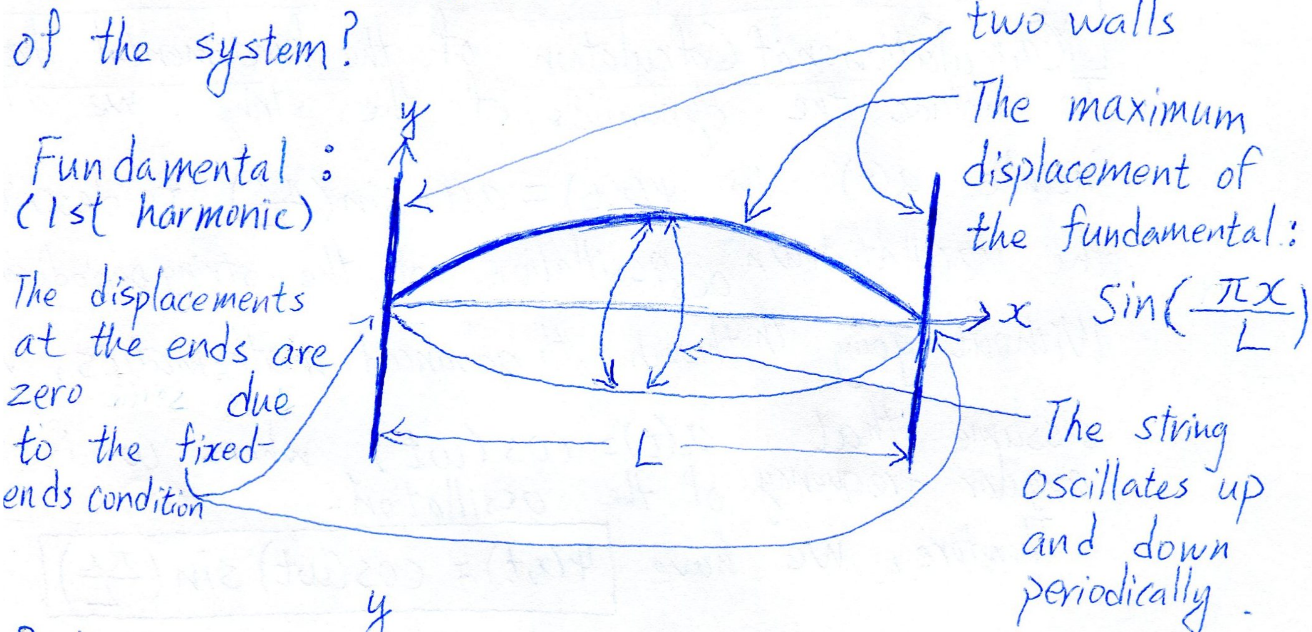
$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\underset{\substack{\uparrow \\ \text{ck law}}}{\left(\frac{\omega}{k}\right)^2}}(-\omega^2 E) = -k^2 E = \frac{\partial^2 E}{\partial x^2}$$

∴ The standard wave form $\sim \cos(kx-\omega t)$ satisfies the expected relation for EM waves in vacuum.

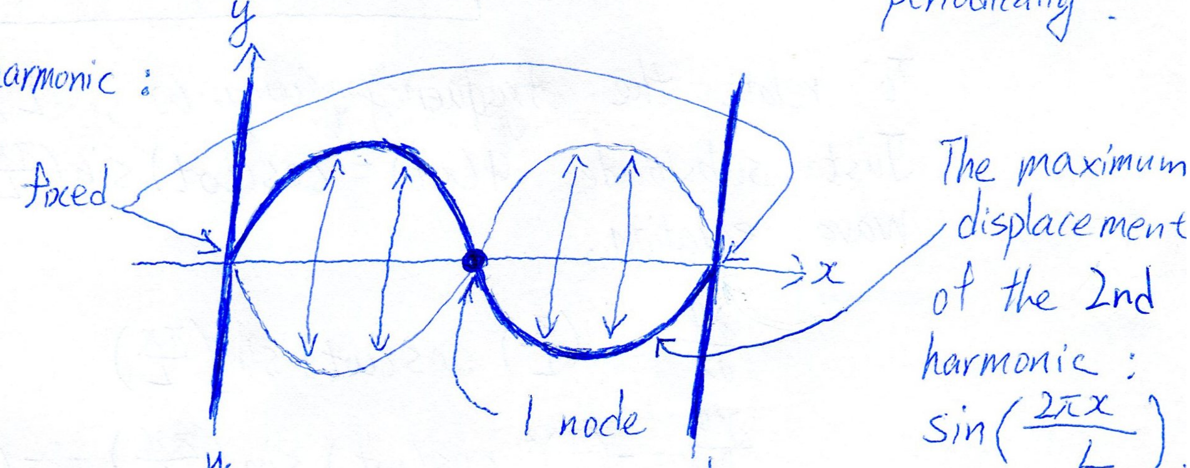
(b) Being different from the travelling wave properties, a proper of this wave is standing due to the fixed-ends condition of the system. Let's recall what you have learnt in your secondary school.

Now we have a string of length L. Let's define T as the tension of the string, and μ as the mass per unit length of the string. How to draw the harmonics of the system?

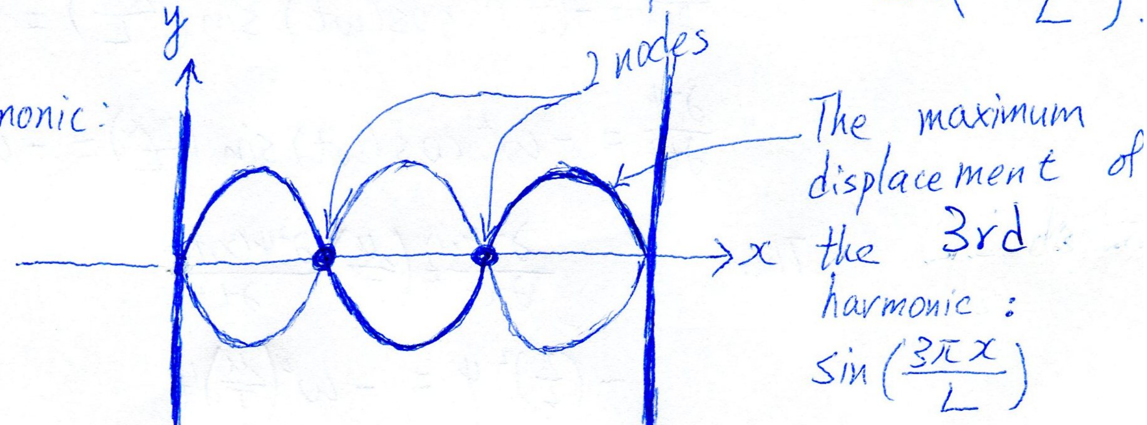
Fundamental:
(1st harmonic)



2nd harmonic:



3rd harmonic:



⋮

From the above drawings, we can conclude that the maximum displacement of the n-th harmonic is $\sin(\frac{n\pi x}{L})$.

Main Question: Calculation of the fundamental frequency

To include the dynamics of the string, we include $a(t)$ in $\psi(x,t) = a(t)\sin(\frac{\pi x}{L})$ to describe the up-and-down ocsillation of the string periodically. Without going through advanced mathematics, we assume that $a(t) = \cos(\omega t)$ where ω is the angular frequency of the ocsillation.

Therefore, we have $\boxed{\psi(x,t) = \cos(\omega t)\sin(\frac{\pi x}{L})}$

To relate the angular frequency ω, L, T and μ. Just substitude $\psi(x,t) = \cos(\omega t)\sin(\frac{\pi x}{L})$. into the wave equation.

$$\frac{\partial \psi}{\partial x} = \left(\frac{\pi}{L}\right)\cos(\omega t) \left(\frac{\pi x}{L}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{\pi}{L}\right)^2 \cos(\omega t)\sin\left(\frac{\pi x}{L}\right) = -\left(\frac{\pi}{L}\right)^2 \psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \cos(\omega t)\sin\left(\frac{\pi x}{L}\right) = -\omega^2 \psi.$$

Then
$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \left(\frac{\mu}{T}\right)\frac{\partial^2 \psi(x,t)}{\partial t^2}$$

$$-\left(\frac{\pi}{L}\right)^2 \psi = -\omega^2\left(\frac{\mu}{T}\right)\psi$$

$$\omega^2 = \frac{T}{\mu}\left(\frac{\pi}{L}\right)^2$$

$$\omega = \sqrt{\frac{T}{\mu}}\,\frac{\pi}{L}$$

The fundamental frequency

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

And the corresponding fundamental wavelength

$= 2L$